



MEDICAL UNIVERSITY – PLOVDIV

FACULTY OF PHARMACY

SYLLABUS

IN

ADVANCED MATHEMATICS

Approved by the Department Council, Protocol №109/28.10.2024

Confirmed by the Faculty Council, Protocol №??/29.10.2024

ADVANCED MATHEMATICS
Syllabus

Discipline	Final exam/ semester	According to the Faculty of Pharmacy curriculum of MU-Plovdiv Academic hours				ECTS	Academic hours in semester			
		Auditorium	Lectures	Practices	Non-auditorium		I semester		II semester	
							L	P	L	P
Advanced Mathematics	1	60	15	45	60	4,0	1	3		

DISCIPLINE:

Advanced Mathematics

TYPE OF DISCIPLINE ACCORDING TO THE UNIFORM STATE REQUIREMENTS:

Compulsory

LEVEL OF QUALIFICATION:

Master

FORMS OF TRAINING:

Lectures, seminars, self-training

YEAR OF TRAINING:

First

DURATION OF TRAINING:

One semester

ACADEMIC HOURS:

15 academic hours of lectures, 45 academic hours of seminars

TECHNICAL EQUIPMENT APPLIED IN THE TRAINING:

Multimedia presentations, discussions, demonstrations of methods for solving mathematical problems, solving specific computing tasks, preparation for theoretical testing.

FORMS OF EVALUATION:

Exam, continuous assessment by two colloquia, doing tests, problem solving.

EVALUATION CRITERIA:

Formed to final evaluation of the exam as an average score for each of the three parts of the material as well as the score of the theoretical online test.

ASPECTS OF EVALUATION CRITERIA:

True, complete, and accurate solving of the tasks and correctly solving of the tests.

SEMESTER EXAM:

Yes (entry test, written examination, and discussion for forming the final mark).

STATE EXAM:

No.

LECTURER:

Professor of Mathematics

DEPARTMENT:

Medical Physics and Biophysics

ANNOTATION

Discipline "Advanced Mathematics" is a fundamental discipline and allows for the acquisition of knowledge and skills required of non-specialists in mathematics, but useful for specialty students. This includes the basic topics such as: Matrices and determinants. Systems of linear equations. Polynomials. Vectors. Equations of a straight line in the plane. Study of a function of one variable using limits and first and second derivatives. Definite, indefinite, improper integrals. Extremes of a function of two variables.

BASIC AIMS OF THE DISCIPLINE

Acquisition of knowledge and skills to implement all methods and tools for solving mathematical problems. The main objective of the educational process in this course is the knowledge of the basic theoretical facts and the learning of methods for solving problems associated with them, such as:

- Operations with matrices
- Calculation of determinants
- Solving systems of linear equations by using the methods of Cramer and Gauss
- Eigenvalues and eigenvectors of a matrix of order 2
- Operations with polynomials and factorization
- Linear and metric vector operations and their application
- Analytical set of a point, a straight line, a circle in the plane
- Implementation of vectors for solving planimetry tasks
- Studying of a function of one variable using limits and derivatives
- Solving basic types of indefinite integrals
- Definite and improper integrals and their application
- Solving ordinary differential equations with separable variables
- Finding the partial derivatives of functions of two variables

- Finding extrema of functions of two variables
- Applications of the studied mathematical apparatus in pharmaceutical sciences

EXPECTED RESULTS

After completion of the course students must have the following knowledge and skills:

- To gain confidence when working on mathematical topics and tasks
- To fill gaps in secondary education in mathematics
- Be familiar with the basic facts of the studied mathematical topics
- Be proficient in proper degree in studied methods for solving the essential tasks
- To know about the main applications of the studied mathematical apparatus in the pharmaceutical sciences
- Be able to use mathematical literature and electronic resources of calculation

LECTURES

PART I. LINEAR ALGEBRA AND ANALYTICAL GEOMETRY

1. Matrices.
 - 1.1. Matrix of $(m \times n)$ -type: definition, types.
 - 1.2. Matrix operations: comparison and addition of matrices, matrix multiplication by a number. Properties of these operations.
 - 1.3. Operations with matrices, matrix multiplication – definition and properties.
2. Determinants.
 - 2.1. Definition of the determinant of the (2×2) and (3×3) matrix; Minor and Adjugate matrix element of the determinant.
 - 2.2. Properties of determinants.
3. The inverse matrix and applications
 - 3.1. Inverse matrix of a matrix – definition.
 - 3.2. Existence and methods for finding
 - 3.2.1. Method of Adjugate matrix.
 - 3.2.2. Method of Gauss-Jordan.
 - 3.3. Application in solving linear matrix equations.
4. Systems of linear equations
 - 4.1. System of m linear equations with n variables – definition; solution of the system; (non-)homogeneous, (in-)consistent systems, (in-)finite solutions.
 - 4.2. Methods for solving systems of linear equations: Kramer's method, through a matrix equation and the method of Gauss.
5. Polynomials
 - 5.1. Polynomial – definitions (indeterminate, term, degree, coefficients, zero polynomial). Equal polynomials.
 - 5.2. Addition, subtraction and multiplication of polynomials. Properties.
 - 5.3. Dividing of polynomials – definition. Simple and multiple polynomial roots, the number of roots of polynomials and presentation of the polynomial by its roots.
 - 5.4. Methods for dividing polynomials: the direct divide; indeterminate coefficients; Horner's method. Examples.
6. Vectors
 - 6.1. Vector, directed line segment, length of the vector, collinear vectors, zero vector, opposite vector of a vector. Actions with vectors and properties. Unit vector. Linearly (in-)dependent vectors, linear combination of vectors.

- 6.2. Cartesian coordinate systems on a line and on a plane. Coordinates of point, a vector, a directed line segment and a linear combination of vectors.
- 6.3. Scalar product of vectors – definition, vanishing, properties, applications. Calculation on a Cartesian coordinate system in the plane and in the space.
- 7. Equations of a line in the plane and conic sections.
 - 7.1. Equations of a line in the plane: vector parametric, scalar parametric, canonical, through two points, general, Cartesian and interceptional.
 - 7.2. Applications of equations of line: angle between two lines, the intersection point of lines.
 - 7.3. Equations of Conic Sections – Second-order curves. Definition of their parameters and their properties. Circle.
- 8. Eigenvalues, eigenvectors, and matrix diagonalization.
 - 8.1. Eigenvalues and eigenvectors of a square matrix of order 2 - spectrum, eigensubspaces.
 - 8.2. Eigenvalues and eigenvectors of a symmetric matrix.
 - 8.3. Diagonalization of a matrix – non-singular and symmetric.

PART II. INVESTIGATION OF FUNCTION OF ONE AND TWO VARIABLE.

- 9. Functions
 - 9.1. Function – definition and ways of setting.
 - 9.2. Types of functions – even, odd, periodic, monotone, limited.
 - 9.3. Reverse and reversible functions – definitional intervals.
 - 9.4. Napier’s constant, natural logarithm, exponential function.
- 10. Limit and continuity of function
 - 10.1. Limit of a function (left, right), limit point. Properties. Some basic limits Indeterminate forms.
 - 10.2. Continuity of function – (dis-)continuous function at a point.
- 11. Derivative and differential of function
 - 11.1. Derivative of function – definition, geometrical and mechanical interpretation.
 - 11.2. Differential of function. Derivatives and differentials of higher order.
- 12. Applications of the derivatives
 - 12.1. L’Hôpital’s theorems for evaluation of the limit of function. Applying for indeterminate forms.
 - 12.2. Representation of a function by Taylor’s and Maclaurin’s series. An example of the exponential function.
 - 12.3. Extrema and investigation of functions of one variable
 - 12.3.1. Criteria for constancy and monotony of function.
 - 12.3.2. Convexity, concavity and inflection points of function.
 - 12.3.3. Local and absolute extrema of function.
 - 12.3.4. Asymptotes.
- 13. Functions of two variables
 - 13.1. Definition of a function of two variables. Total and partial rates of change. Limit and continuity of a function of two variables.
 - 13.2. Partial derivatives of first and second order.
 - 13.3. Local extrema of a function of two variables.

PART III. INTEGRALS.

- 14. Indefinite integrals

- 14.1. Indefinite integrals, primitive function (antiderivative), basic rules for integration.
- 14.2. Integration by importing under the sign of the differential; by substitution.
- 14.3. Integration of a rational function; of a trigonometric function.
- 15. Definite integrals and improper integrals
 - 15.1. Definite integrals – definition and geometric interpretation.
 - 15.2. Basic properties of definite integrals.
 - 15.3. Basic methods for calculating definite integrals, change of variable
 - 15.4. Types of improper integrals and applications.
 - 15.5. Ordinary differential equations of first order: with separable variables, homogeneous, linear.

PRACTICES

- 1. Matrices and determinants
 - 1.1. Determinant of the second order
 - 1.2. Determinant of the third order
 - 1.3. Determinant of fourth and higher order
 - 1.4. Linear operations with matrices. Transposition
 - 1.5. Matrix multiplication
 - 1.6. Inverse matrix
 - 1.7. Matrix equations
- 2. Systems of linear equations
 - 2.1. Cramer's rule for solving
 - 2.2. Gaussian elimination for solving
- 3. Polynomials
 - 3.1. Sum, difference, product and quotient of polynomials
 - 3.2. Decomposition of polynomial multipliers
 - 3.3. Decomposition rational expression of elementary fractions
- 4. Cartesian coordinate systems
 - 4.1. Cartesian coordinate system on the axis
 - 4.2. Cartesian coordinate system in the plane
- 5. Equations in the plane
 - 5.1. Incidence of point and rights
 - 5.2. Types of line equations
 - 5.3. Lines through point and direction
 - 5.4. Angle between lines
- 6. Equations of Conic Sections – Second-order curves.
 - 6.1. Definition of their parameters and their properties.
 - 6.2. Circle.
- 7. Eigenvalues, eigenvectors, and matrix diagonalization.
 - 7.1. Eigenvalues and eigenvectors of a square matrix of order 2 - spectrum, eigensubspaces.
 - 7.2. Eigenvalues and eigenvectors of a symmetric matrix.
 - 7.3. Diagonalization of a matrix – non-singular and symmetric.
- 8. Types of functions
 - 8.1. The domain of function
 - 8.2. Monotony of function
 - 8.3. Odd and even functions

- 8.4. Periodic functions
- 9. Derivative and differential of function
 - 9.1. First derivative of function
 - 9.2. First differential of function
 - 9.3. Derivatives of higher-order of a function
 - 9.4. Differentials of higher-order of a function
- 10. Calculation of the limit of a function
 - 10.1. By transformation of rational expression
 - 10.2. In indeterminate form $[0:0]$ or $[\infty:\infty]$
 - 10.3. In indeterminate form $[0.\infty]$
 - 10.4. In indeterminate form $[\infty - \infty]$
- 11. Extrema and investigation of functions of one variable
 - 11.1. Extrema of function of one variable
 - 11.2. Convexity and concavity of a function of one variable
 - 11.3. Asymptotes of function of one variable
 - 11.4. Investigation of function of one variable
- 12. Functions of two variables. Local extrema.
- 13. Indefinite integrals of function
 - 13.1. Basic methods of integration
 - 13.2. Integration of rational functions
- 14. Definite and improper integrals of function
 - 14.1. Definite integrals
 - 14.2. Improper integrals
- 15. Ordinary differential equations of first order
 - 15.1. Separable
 - 15.2. Homogeneous
 - 15.3. Linear

BIBLIOGRAPHY

1. **M. Manev, H. Manev.** *Textbook of Advanced Mathematics for pharmaceutical students at the Medical University of Plovdiv*, Macros, Plovdiv, 2022. (ISBN 978-954-561-572-6)
2. **M. Manev, H. Manev.** *Handbook of Advanced Mathematics for pharmaceutical students at the Medical University of Plovdiv*, Macros, Plovdiv, 2022. (ISBN 978-954-561-570-2)

Can be used to some extent and other textbooks, guide-books and collections of tasks which are addressed to non-mathematical students from different universities.

CONSPECTUS

Part I.

1. Definition of the determinant of the second and third order; minor and adjugate matrix element of the determinant.
2. Properties of determinants.
3. Matrix of type $m \times n$: definition, types.
4. Matrix operations: comparison and addition of matrices, matrix multiplication by a number. Properties of these operations.
5. Operations with matrices, matrix multiplication – definition and properties.
6. Inverse matrix of a matrix – definition, existence and methods for finding (of the adjugate matrix and the Gauss-Jordan method, application in solving linear matrix equations.
7. System of m linear equations with n variables – definition; decision of the system; (in-) homogeneous, (in-) compatible, (in-) determinate systems.
8. Methods for solving systems of linear equations: Cramer's, by a matrix equation and the method of Gauss.
9. Polynomial – definitions (argument, term, degree, coefficients, zero polynomial). Equal polynomials.
10. Addition, subtraction and multiplication of polynomials. Properties.
11. Division of polynomials – definition. Simple and multiple zero of the polynomial, the number of zeros of polynomials and representation of the polynomial by its zeros.
12. Methods for dividing polynomials: the direct division; of indeterminate coefficients; Horner's. Examples.
13. Vector, directed line segment, length of the vector, collinear vectors, zero vector, opposite vector of a vector. Operations with vectors and properties. Unit vector. Linearly (in-) dependent vectors, linear combination of vectors.
14. Cartesian coordinate systems on the line, in the plane and in space. Coordinates of the point, of the vector, directed line segment and a linear combination of vectors.
15. Scalar product of vectors – definition, vanishing, properties, applications. Calculation on a Cartesian coordinate system in the plane and in space.
16. Equations of a line in the plane: vector parametric, scalar parametric, canonical, through two points, general, Cartesian and interceptional.
17. Applications of equations of lines: angle between two lines, the intersectional point of lines.
18. Equations of Conic Sections – Second-order curves. Definition of their parameters and their properties. Circle.
19. Eigenvalues, eigenvectors, and matrix diagonalization. Eigenvalues and eigenvectors of a square matrix of order 2 - spectrum, eigensubspaces. Eigenvalues and eigenvectors of a symmetric matrix. Diagonalization of a matrix – non-singular and symmetric.

Part II.

20. Function - definition and ways of setting. Types of functions - even, odd, periodic, monotonic, limited.
21. Reverse and reversible functions - definitional intervals. Napier's constant, natural logarithm, exponential function.
22. Limit of a function (left, right), limit point. Properties. Some basic limits. Indeterminate forms. Asymptotes.
23. Continuity of function - (dis-) continuous function at a point.
24. Derivative of a function - definition, geometric and mechanical interpretation.
25. Differential function. Derivatives and differentials of higher order.
26. L'Hôpital's theorems for evaluation of the limit of function. Applying for indeterminate forms.
27. Representation of a function by Taylor's and Maclaurin's series. An example of the exponential function An example of the exponential function.
28. Criteria for constancy and monotony of function.
29. Convexity, concavity, and inflection point of the function.
30. Local and global extrema of the function.
31. Functions of two variables. Total and partial rates of change. Limit and continuity of a function of two variables. Partial derivatives of first and second order.
32. Local extrema of a function of two variables.

Part III.

33. Indefinite integrals, primitive function (antiderivative), basic rules for integration.
34. Integration by importing under the sign of the differential; by substitution.
35. Integrating a rational function; of trigonometric function.
36. Definite integrals – definition and geometric interpretation.
37. Basic properties of definite integrals.
38. Main theorems for definite integrals: for mean value; for integration by parts; by substitution.
39. Improper integral. Applications.
40. Ordinary differential equations of first order: with separable variables, homogeneous, linear.